# ALGORITHM FOR FACTOR ANALYSIS IN INVERSIVE SPACE

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***Summary***
*The aim of this research was to determine possibilities of latent dimension explication in inversive space which is usually completely ignored as a systematic and different mistake. Research design was made for PC by Borland Delphi developing tool. For the needs of this research and example, data for students described by 14 morphological variables have been processed. The results have shown that inversive space defined this way is not chaotic and that it possesses real existence which was possible to recognize and comprehend, and therefore credibly explain. Restrictions of models and algorithms only exist in the situation when inversion matrices of initial data are not Gramian, and this procedure is even possible in positive self-definite matrices. Practical implications of this research are more than explicit since they enable more detailed research on phenomena which were until now trapped in the space of unknown without possibilities of scientific clarification. Original way of research is indisputable, because there are simply no similar algorithms.*

***Key words****: algorithm, inversive, factorization.*

## INTRODUCTION

Analysis of existence, content, information contribution and structure of latent dimensions has always been a real challenge for researchers in everyday scientific discipline that deals with exploring the unknown part of variability in research developments (Rao, 1973; Fulgosi, 1979; Bonacin, 2004).
Latent dimensions which are used to describe such developments are almost always defined on the data that are somehow related (e.g. correlations), and therefore information that we can easily recognize as latent dimensions are extracted through them with the help of appropriate procedures (Harmanm 1970; Mulaik, 1972; Momirovic, 1984). Although it all usually “takes place” in so called realistic space, for the purpose of identification of additional phenomena, it is possible to set the metric of variables in other spaces such as Guttman Image, Harris space….(Cooley & Lochnes, 1971; Momirovic, 1984; Johnson & Wicher, 1992; Bonacin, 2004; Bonacin, 2006).
Sure that given results, although initially on the same data, will not be equal if different procedures are applied for defining the metric of variables. It is usually possible to determine certain mathematic relations between different sets of latent dimensions under different metrics. However, there is a metric which is never used, and which maybe has its point. In other words, in determining joint part of variability of variable set or in defining one based on set of the remaining ones (smc, h2), the most effective procedure for determining uniquity is inversion of the matrix of relations, where the joint part (communality) of variables gets inversive matrices of relation through a large diagonal. At the same time, that doesn’t mean that it is about completely different space, which is possible to be used on one completely different way.

### Model

Information on the inversive dimension of some sample described by a set of, in principle (although not exclusively) quantitative variables which are interested for two reasons:
1) determining the part of variability which slowly slips from our hands and observing (if we look everything from the point of objective and positivistic rationality), & 2) inversive space is not particularly looked into, and therefore it is not very recognized. For the sake of easier understanding, it could be said, for example in terminology of physical technology, that for example relations between elements of some matter can be studied as firmness and inflexibility, that is resistance to primary disintegration, in other words preserving integrity. In that case, inversive space is in essence possibility of deformation, and consequently possibility of elasticity and plasticity as well (if the

force of effect is too high to overcome internal compactness of material). So, inversive space will indeed be opposite to the initial one, which therefore means that we can also study it on equal terms, and not to disregard it as the unknown part of variability. It will include parts of variability for which inversive latent dimensions are responsible.

**Algorithm**

Let's say that by using some criterion of definition, any object sample was extracted from E population with condition of representation e**i** *(i=1..x**1**)* described by set of variables v**j** *(j=1..y)* extracted from V population. By Hadamar method of value joining using selected objects we can get matrix of gross data B = E ⊗ V with elements b**i,j** *(i=1..x, j=1..y)*. Let's now standardize data in matrix B so that we have z**ij** = (b**ij** – m **j**) / σ**j**, where the vector of arithmetic mean is contained in m=Σb**1j** / x, while the standard deviation is defined as σ**1** = sqrt(Σ(b**ij**-m**j** $)^2$ / x). In matrix form, from the data standardized this way, we get correlations of variables under model of the highest authenticity such as R = Z' Z **1**/**x.** Using the operation of inversion under conditions I = R**1** * R$\mathbf{1}^{-1}$, we calculate diagonal elements (such as D**jj** = dg (R**1**$)^{-)}$ ) from R matrix, which presents evaluation of one part of unexplained variability of all variables separately evaluated on the basis of the remaining ones, that is variable uniquity ($u^2$**j**).

Sure, communality of variables from R1 and R2 are simply $h^2$**j** = 1 **-** $u^2$**j**. Vector of communality $h^2$**j** presents explained part of variability for each and every variable on the basis of the remaining ones. At this moment in the procedures of analysis, inversive space is usually given up as uninteresting. However, if we take inversive matrix (R$\mathbf{1}^{-1}$) we can see that it is positively definite, although there are situations where it is positively semi-definite and negatively definite, which depends on quality of the project in which it was extracted. In any case, if it is really positively definite (which is checked through determinant, distinctive values and communality of the original matrix), it is possible to apply the following procedure:

1. define diagonal matrix C=1/sqrt(dg(R$\mathbf{1}^{-1}$));
2. operation A = C * R$\mathbf{1}^{-1}$ * C, pre- and post-multiplication extract matrix A;
3. we now treat matrix A as a correlation of inversive variables in the system;
4. further procedure like the one with authentic data gives inversive latent dimensions.

## EXAMPLE, METHODS AND RESULTS

For the model illustration, and for the needs of this work, we chose the sample group of 249 male students who are described by 14 morphological variables from very beginning of the first grade of elementary school, and measurement was made within project "Effectiveness of kinetic treatment for ages from 7 to 10" (MZT RH: 5-10-218). Data from three rounds of measurement were concatenated. Applied variables were: : body height, leg length, arm length, wrist diameter, knee diameter, biacromial width, bicrystal width, body weight, forearm circumference, lower leg circumference, average chest circumference, upper arm skin fold, back skin fold, and abdominal skin fold. These variables covered the whole morphological space within longitudinal transversality, volume and fatty tissue.

*Table 1. Aslope rotated unit of Promac factor in the initial space and correlations of factors*

| | PRX1 | PRX2 |
|---|---|---|
| body height | **0.95** | -0.18 |
| leg length | **0.91** | -0.19 |
| arm length | **0.88** | -0.21 |
| wrist diameter | **0.76** | 0.02 |
| knee diameter | **0.72** | 0.08 |
| biacromial width | **0.83** | -0.07 |
| bicrystal width | **0.74** | 0.09 |
| body weight | **0.80** | 0.31 |
| forearm circumference | **0.62** | 0.34 |
| lower leg circumference | **0.65** | 0.37 |
| average chest circumference | **0.58** | 0.47 |
| upper arm skin fold | -0.05 | **0.89** |
| back skin fold | -0.11 | **0.95** |
| abdominal skin fold | -0.07 | **0.94** |
| | **PRX1** | **PRX2** |
| PRX1 | **1.00** | **0.34** |
| PRX2 | | **1.00** |

*Table 2. Aslope rotated unit of Promac factor in the inversive space and correlations of factors*

| | PRX1 | PRX2 | PRX3 | PRX4 | PRX5 | PRX6 | PRX7 | PRX8 |
|---|---|---|---|---|---|---|---|---|
| **body height** | 0.06 | **0.90** | -0.14 | -0.03 | -0.17 | -0.01 | 0.01 | -0.20 |
| **leg length** | -0.02 | **-0.85** | -0.06 | -0.02 | -0.03 | -0.05 | -0.01 | -0.49 |
| **arm length** | -0.12 | 0.07 | 0.12 | 0.02 | 0.03 | -0.10 | -0.04 | **0.88** |
| **wrist diameter** | -0.03 | -0.11 | 0.19 | 0.07 | -0.17 | **-0.79** | -0.18 | -0.03 |
| **knee diameter** | -0.13 | -0.07 | 0.18 | 0.06 | -0.16 | **0.81** | 0.02 | -0.16 |
| **biacromial width** | 0.03 | 0.01 | 0.06 | 0.02 | -0.05 | 0.16 | **0.86** | -0.02 |
| **bicrystal width** | 0.12 | -0.14 | -0.15 | -0.03 | **-0.60** | 0.20 | -0.46 | 0.27 |
| **body weight** | -0.12 | -0.16 | -0.15 | -0.02 | **0.93** | 0.05 | -0.13 | 0.11 |
| **forearm circumference** | -0.37 | 0.03 | **-0.71** | 0.00 | -0.29 | -0.11 | 0.07 | -0.05 |
| **lower leg circumference** | -0.13 | -0.07 | **0.81** | -0.06 | -0.26 | -0.06 | 0.12 | 0.13 |
| **average chest circumference** | **0.77** | 0.18 | 0.12 | 0.01 | -0.05 | -0.02 | -0.26 | -0.28 |
| **upper arm skin fold** | **0.75** | -0.11 | -0.10 | -0.03 | -0.18 | -0.08 | 0.39 | 0.11 |
| **back skin fold** | -0.41 | 0.09 | 0.04 | **-0.85** | -0.03 | 0.02 | -0.08 | -0.04 |
| **abdominal skin fold** | -0.36 | 0.07 | -0.03 | **0.87** | -0.05 | 0.01 | -0.05 | -0.01 |
| | **PRX1** | **PRX2** | **PRX3** | **PRX4** | **PRX5** | **PRX6** | **PRX7** | **PRX8** |
| **PRX1** | 1.00 | -0.09 | 0.00 | -0.01 | 0.19 | 0.08 | 0.04 | 0.11 |
| **PRX2** | | 1.00 | 0.08 | -0.06 | 0.05 | -0.04 | 0.03 | -0.12 |
| **PRX3** | | | 1.00 | 0.04 | 0.10 | -0.05 | -0.07 | 0.01 |
| **PRX4** | | | | 1.00 | -0.05 | 0.05 | -0.09 | -0.06 |
| **PRX5** | | | | | 1.00 | 0.01 | -0.04 | 0.01 |
| **PRX6** | | | | | | 1.00 | -0.07 | 0.06 |
| **PRX7** | | | | | | | 1.00 | 0.03 |
| **PRX8** | | | | | | | | 1.00 |

## DISCUSSION AND CONCLUSION

According to Table 1. we obtained 2 latent dimensions which were easy to identify as: 1. general mechanism of growth and development of active locomotor segments (bones, muscle system); 2. mechanism of growth and development of passive ballast tissues (fatty tissue). These two latent mechanisms are in significant but not so high relations (0.34). It can be concluded that development of students is still largely undifferentiated , that is differentiation which can be recognized in grown men has not yet started, therefore there is still no separation inside the bone development on longitudinality and transversality, while at the same time integration of muscle system in the active syncretistic system is still present.

However, in inversive space (Table 2.), many interesting details can be seen. First of all, that's how no less than 8 relatively independent latent dimensions were isolated. Therefore if integrative processes are registered in the initial space and realistic metrics, in that case the word natural means that disintegrative phenomena can be expected in several (8) ways, where each of them has its characteristics (Cote & Buckley, 1987; Momirovic & sur., 1987; Bonacin, 2004). It is visible that the volume of thoracic cavity and the fold of upper arm go in the same direction (Prx1), and other folds are negative, which presents the evidence of existence of some sub-mechanism of upper body voluminosty. Noticable values of leg height and length can also be seen. Although being very high, these values are with opposite signs (Prx2), which says that there is sub-mechanism of height growth and leg growth divergence. Very similarly, but very topologically, it can also be noticed in lower leg and forearm volume (Prx3), which means that mechanism of uneven development of distal extremity segments. That situation is repeated with fatty tissue (Prx4) which describes topological difference of fatty tissue development in relation to frontal flat surface.Especially interesting is the position of bicrystal range and body mass (Prx5) where it can be seen that one of the directions of disintegration is directed towards mass in relation to body center. Similar to mechanism Prx3, transversal dimensions (Prx6) are sensible to topological functionality, so one can recognize topological divergence of joint development. The last two factors (Prx7 and Prx8) describe only one independent segment of development divergence, specifically biacromial and the growth and development of arm length. As it can be seen, analysis in inversive space gave entirely logical indicators and refers to multiple divergences which can not be detected by other methods. It can be concluded that problems with latent dimensions must not on any account be strictly seen in the initial space, regardless of the fact is it realistic, image, Hariss or something like that. It is obvious that models constructed under conditions of explication of latent phenomena from variable relation can not give complete answers on many questions concerning determining of the phenomena that can not be directly measured, regardless of the fact which scientific discipline we talk about. Part of the answer surely lies in inversion metrics, and as far as we can see, based on examples, many interesting findings and conclusions are possible, logical and potentially very interesting for the scientific world. It is not out of question that this method will become very poplar, after its good sides are determined, therefore it can be recommended as a very strong complement procedure in all analyses of latent dimensions, regardless of which changes we talk about - explorative, confirmative or other.

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## ALGORITAM ZA FAKTORSKU ANALIZU U INVERZNOM PROSTORU

*Originalni naučni rad*


**Sažetak**

Svrha ovog istraživanja bilo je utvrđivanje mogućnosti eksplikacije latentnih dimenzija u inverznom prostoru koji se obično potpuno zanemaruje kao sistematska i drugačija greška. Dizajn istraživanja bio je usmjeren na logički model i matematički algoritam koji je kao računalni program je realiziran za PC računalo u razvojnom alatu Borland Delphi. Za potrebe ovog istraživanja i primjer obrađeni su podaci učenika opisanih sa 14 morfoloških varijabli. Rezultati su pokazali da inverzni prostor definiran na ovaj način nije kaotičan i da posjeduje stvarnu egzistenciju koju je bilo moguće prepoznati i razumjeti, pa time i vjerodostojno interpretirati. Ograničenja modela i algoritma samo postoje u situacijama kad matrice inverza inicijalnih podataka nisu gramianske, a čak je postupak moguć i kod pozitivno semidefinitnih matrica. Praktične implikacije ovog istraživanja više nego su jasne jer omogućavaju detaljnije istraživanje fenomena koji su do sada ostajali u prostoru nepoznatog bez mogućnosti znanstvenog rasvijetljavanja. Originalnost istraživanja je nedvojbena, jer sličnih algoritama jednostavno nema.


**Ključne riječi**: algoritam, inverz, faktorizacija


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